EXAMPLES

TOPIC :- A

*** Prove that : (સાબિત કરો)**

1. $\log \frac{51}{80} + \log \frac{44}{85} - \log \frac{99}{160} - \log \frac{8}{15} = 0$

$$\text{L.H.S.} = \log \frac{51}{80} + \log \frac{44}{85} - \log \frac{99}{160} - \log \frac{8}{15}$$

$$= \log \left(\frac{51}{80} \times \frac{44}{85} \div \frac{99}{160} \div \frac{8}{15} \right)$$

(∵ લઘુગણક ના ગુણાકાર અને ભાગાકાર ના નિયમો)

$$= \log \left(\frac{51}{80} \times \frac{44}{85} \times \frac{160}{99} \times \frac{15}{8} \right)$$

$$= \log \left(\frac{17 \times 3}{80} \times \frac{11 \times 4}{17 \times 5} \times \frac{80 \times 2}{11 \times 3 \times 3} \times \frac{5 \times 3}{4 \times 2} \right)$$

$$= \log 1$$

$$= 0 = \text{R.H.S.}$$

2. $2 \log \frac{6}{7} + \frac{1}{2} \log \frac{81}{16} - \log \frac{27}{196} = \log 12$

$$\text{L.H.S.} = 2 \log \frac{6}{7} + \frac{1}{2} \log \frac{81}{16} - \log \frac{27}{196}$$

$$= \log \left(\frac{6}{7} \right)^2 + \log \left(\frac{81}{16} \right)^{1/2} - \log \frac{27}{196}$$

$(\because \log x^n = n \log x)$

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$$= \log \frac{36}{49} + \log \frac{9}{4} - \log \frac{27}{196}$$

$$= \log \left(\frac{36}{49} \times \frac{9}{4} \div \frac{27}{196} \right)$$

$$= \log \left(\frac{36}{49} \times \frac{9}{4} \times \frac{196}{27} \right)$$

$$= \log \left(\frac{9 \times 4}{7 \times 7} \times \frac{3 \times 3}{4} \times \frac{14 \times 14}{9 \times 3} \right)$$

$$= \log (3 \times 2 \times 2)$$

$$= \log 12 = \text{R.H.S.}$$

3. $\log \frac{15}{7} - \log \frac{25}{4} + \log \frac{35}{12} = 0$

$$\text{L.H.S.} = \log \frac{15}{7} - \log \frac{25}{4} + \log \frac{35}{12}$$

$$= \log \left(\frac{15}{7} \div \frac{25}{4} \times \frac{35}{12} \right)$$

$$= \log \left(\frac{15}{7} \times \frac{4}{25} \times \frac{35}{12} \right)$$

$$= \log \left(\frac{5 \times 3}{7} \times \frac{4}{5 \times 5} \times \frac{7 \times 5}{3 \times 4} \right)$$

$$= \log 1$$

$$= 0 = \text{R.H.S.}$$

4. $\log\left(\frac{x^a}{x^b}\right)+\log\left(\frac{x^b}{x^c}\right)+\log\left(\frac{x^c}{x^a}\right)=0$

$$\text{L.H.S.} = \log\left(\frac{x^a}{x^b}\right)+\log\left(\frac{x^b}{x^c}\right)+\log\left(\frac{x^c}{x^a}\right)$$

$$= \log\left(\frac{x^a}{x^b}\times\frac{x^b}{x^c}\times\frac{x^c}{x^a}\right)$$ [∵ લઘુગણક માટે ગુણાકાર નો નિયમ]

$$= \log\left(x^{a-a}\times x^{b-b}\times x^{c-c}\right)$$

$$= \log\left(x^0\times x^0\times x^0\right)$$

$$= \log(1\times1\times1)$$

$$= \log 1$$

$$= 0 = \text{R.H.S.}$$

5. $\log(\log x^2)-\log(\log x)=\log 2$

$$\text{L.H.S.} = \log(\log x^2)-\log(\log x)$$

$$= \log\left(\frac{\log x^2}{\log x}\right)$$ [$\because \log a-\log b=\log\frac{a}{b}$]

$$= \log\left(\frac{2\log x}{\log x}\right)$$

$$= \log 2 = \text{R.H.S.}$$

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6. $\log\left(x+\sqrt{x^2-1}\right)+\log\left(x-\sqrt{x^2-1}\right)=0$

L.H.S. $=\log\left(x+\sqrt{x^2-1}\right)+\log\left(x-\sqrt{x^2-1}\right)$

$$=\log\left[\left(\overset{a}{x}+\overset{b}{\sqrt{x^2-1}}\right)\left(\overset{a}{x}-\overset{b}{\sqrt{x^2-1}}\right)\right]$$

$(\because \log a+\log b=\log ab)$

$$=\log\left(x^2-(x^2-1)\right) \quad \left[\because (a+b)(a-b)=a^2-b^2\right]$$

$$=\log\left(x^2-x^2+1\right)$$

$$=\log 1$$

$$=0=\text{R.H.S.}$$

7. $\log\left(\sqrt{x^2+1}+x\right)+\log\left(\sqrt{x^2+1}-x\right)=0$

L.H.S. $=\log\left(\sqrt{x^2+1}+x\right)+\log\left(\sqrt{x^2+1}-x\right)$

$$=\log\left[\left(\underset{a}{\sqrt{x^2+1}}+\underset{b}{x}\right)\left(\underset{a}{\sqrt{x^2+1}}-\underset{b}{x}\right)\right]$$

$$=\log\left(x^2+1-x^2\right)$$

$$=\log 1$$

$$=0=\text{R.H.S.}$$

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8. $\log_b a \times \log_c b \times \log_a c = 1$

$$\text{L.H.S.} = \log_b a \times \log_c b \times \log_a c$$

$$= \frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}$$ (∵ આધાર - પરિવર્તન નો નિયમ)

$$= 1 \times 1 \times 1$$

$$= 1 = \text{R.H.S.}$$

9. $\log_{b^3} a^2 \cdot \log_{c^3} b^2 \cdot \log_{a^3} c^2 = \frac{8}{27}$

$$\text{L.H.S.} = \log_{b^3} a^2 \cdot \log_{c^3} b^2 \cdot \log_{a^3} c^2$$

$$= \frac{\log a^2}{\log b^3} \times \frac{\log b^2}{\log c^3} \times \frac{\log c^2}{\log a^3}$$

(∵ આધાર - પરિવર્તન નો નિયમ)

$$= \frac{2\log a}{3\log b} \times \frac{2\log b}{3\log c} \times \frac{2\log c}{3\log a}$$

(∵ $\log x^n = n \log x$)

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$$

$$= \frac{8}{27} = \text{R.H.S.}$$

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10. $\dfrac{1}{\log_6 24} + \dfrac{1}{\log_{12} 24} + \dfrac{1}{\log_8 24} = 2$

$$L.H.S. = \frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24}$$

$$= \log_{24} 6 + \log_{24} 12 + \log_{24} 8$$

$$= \log_{24} (6 \times 12 \times 8)$$

$$= \log_{24} 576$$

$$= \log_{24} 24^2$$

$$= 2 \log_{24} 24$$

$$= 2 \times 1 \quad (\because \log_a a = 1)$$

$$= 2 = R.H.S.$$

$$\Rightarrow \log_b a = \frac{\log a}{\log b} = \frac{1}{\frac{\log b}{\log a}} = \frac{1}{\log_a b}$$

નોંધ: પરિક્ષા માં કારણ તરીકે અવશ્ય લખવું.

11. $\dfrac{1}{\log_{12} 60} + \dfrac{1}{\log_{15} 60} + \dfrac{1}{\log_{20} 60} = 2$

$$L.H.S. = \frac{1}{\log_{12} 60} + \frac{1}{\log_{15} 60} + \frac{1}{\log_{20} 60}$$

$$= \log_{60} 12 + \log_{60} 15 + \log_{60} 20$$

$$\left(\because \log_b a = \frac{1}{\log_a b}\right) \Leftarrow \text{ઉપર મુજબ}$$

$= \log_{60}(12 \times 15 \times 20)$

$= \log_{60} 3600$

$= \log_{60} 60^2$

$= 2 \log_{60} 60$

$= 2 \times 1$

$= 2 = R.H.S.$

12. $\dfrac{1}{\log_{ab} abc} + \dfrac{1}{\log_{bc} abc} + \dfrac{1}{\log_{ca} abc} = 2$

$L.H.S. = \dfrac{1}{\log_{ab} abc} + \dfrac{1}{\log_{bc} abc} + \dfrac{1}{\log_{ca} abc}$

$= \log_{abc} ab + \log_{abc} bc + \log_{abc} ca$

$\left(\because \log_b a = \dfrac{1}{\log_a b}\right)$ Ref. Ex. 10

$= \log_{abc}(ab \times bc \times ca)$

$= \log_{abc}(abc)^2$

$= 2 \log_{abc} abc = 2 \times 1 = 2 = R.H.S.$

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13. $\dfrac{1}{\log_x yz+1}+\dfrac{1}{\log_y zx+1}+\dfrac{1}{\log_z xy+1}=1$

L.H.S. $=\dfrac{1}{\log_x yz+1}+\dfrac{1}{\log_y zx+1}+\dfrac{1}{\log_z xy+1}$

$=\dfrac{1}{\log_x yz+\log_x x}+\dfrac{1}{\log_y zx+\log_y y}+\dfrac{1}{\log_z xy+\log_z z}$

$=\dfrac{1}{\log_x xyz}+\dfrac{1}{\log_y xyz}+\dfrac{1}{\log_z xyz}$

$=\log_{xyz} x+\log_{xyz} y+\log_{xyz} z$

$=\log_{xyz} xyz$ $\left(\because \log_b a=\dfrac{1}{\log_a b}\right)$

Ref. Ex. 10

$=1=$ R.H.S.

14. $\log_2\left(\dfrac{1}{320}\right)+\dfrac{1}{\log_{10} 2}+5=0$

L.H.S. $=\log_2\left(\dfrac{1}{320}\right)+\dfrac{1}{\log_{10} 2}+5$

$=\log_2\left(\dfrac{1}{320}\right)+\log_2 10+5$

$\left(\because \log_b a=\dfrac{1}{\log_a b}\right)$

$= \log_2 \left(\frac{1}{320} \times 10 \right) + 5$

$= \log_2 \left(\frac{1}{32} \right) + 5$

$= \log_2 \left(\frac{1}{2^5} \right) + 5$

$= \log_2 2^{-5} + 5$

$= -5 \log_2 2 + 5$

$= (-5 \times 1) + 5$

$= -5 + 5$

$= 0 = \text{R.H.S.}$

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TOPIC : B

1. If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then Prove that $a = b$.

$\Rightarrow$ Given $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$$\therefore\ 2\log\left(\frac{a+b}{2}\right) = \log ab$$

$[\because \log a + \log b = \log ab]$

$$\therefore\ \log\left(\frac{a+b}{2}\right)^2 = \log ab$$

$[\because \log x^n = n \log x]$

$$\therefore\ \left(\frac{a+b}{2}\right)^2 = ab$$

$[\because \log a = \log b$
$\Downarrow$
$\therefore a = b]$

Log is one - one function.
(એક - એક વિધેય)

$$\therefore\ \frac{a^2 + 2ab + b^2}{4} = ab$$

$$\therefore\ a^2 + 2ab + b^2 = 4ab$$

$$\therefore\ a^2 + 2ab - 4ab + b^2 = 0$$

$$\therefore\ a^2 - 2ab + b^2 = 0$$

$$\therefore\ (a-b)^2 = 0$$

$$\therefore\ a - b = 0 \implies \therefore \boxed{a = b}$$

2. If $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then Prove that $\frac{a}{b} + \frac{b}{a} = 6$.

$\Rightarrow$ Given $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$$\therefore\ 2\log\left(\frac{a-b}{2}\right) = \log ab$$

$$\therefore\ \log\left(\frac{a-b}{2}\right)^2 = \log ab$$

$$\therefore\ \left(\frac{a-b}{2}\right)^2 = ab$$ [∵ Log is one-one function]

$$\therefore\ \frac{a^2 - 2ab + b^2}{4} = ab$$

$$\therefore\ a^2 - 2ab + b^2 = 4ab$$

$$\therefore\ a^2 + b^2 = 4ab + 2ab$$

$$\therefore\ a^2 + b^2 = 6ab \quad \text{——— (A)}$$

નોંધ: Exam માં આ Example માં $a^2 + b^2 = 6ab$ Prove કરવાનું પૂછે તો પરિણામ (A) સુધી Example ને Solve કરવો.

All terms are divided By ab.

$$\therefore\ \frac{a^2}{ab} + \frac{b^2}{ab} = \frac{6ab}{ab}$$

$$\therefore\ \boxed{\frac{a}{b} + \frac{b}{a} = 6}$$

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3. If $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$ then Prove that $\frac{a}{b} + \frac{b}{a} = 7$.

$\Rightarrow$ Given $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$

$\therefore 2\log\left(\frac{a+b}{3}\right) = \log ab$

$\therefore \log\left(\frac{a+b}{3}\right)^2 = \log ab$

$\therefore \left(\frac{a+b}{3}\right)^2 = ab$ [$\because$ Log is one - one function]

$\therefore \frac{a^2+2ab+b^2}{9} = ab$

$\therefore a^2 + 2ab + b^2 = 9ab$

$\therefore a^2 + b^2 = 9ab - 2ab$

$\therefore a^2 + b^2 = 7ab$ —— (A)

All terms are divide By ab.

$\therefore \frac{a^2}{ab} + \frac{b^2}{ab} = \frac{7ab}{ab}$

$\therefore \boxed{\frac{a}{b} + \frac{b}{a} = 7}$

4. If $\log\left(\frac{x-y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then Prove that $x^2 + y^2 = 11xy$.

$\Rightarrow$ Given $\log\left(\frac{x-y}{3}\right) = \frac{1}{2}(\log x + \log y)$

$\therefore\ 2\log\left(\frac{x-y}{3}\right) = \log xy$

$\therefore\ \log\left(\frac{x-y}{3}\right)^2 = \log xy$

$\therefore\ \left(\frac{x-y}{3}\right)^2 = xy$ [$\because$ Log is one-one function]

$\therefore\ \frac{x^2 - 2xy + y^2}{9} = xy$

$\therefore\ x^2 - 2xy + y^2 = 9xy$

$\therefore\ x^2 + y^2 = 9xy + 2xy$

$\therefore\ \boxed{x^2 + y^2 = 11xy}$

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5. If $\log(x+y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y$ then Prove that $x^2+y^2 = 7xy$.

$\Rightarrow$ Given $\log(x+y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y$

$\therefore \log(x+y) - \log 3 = \frac{1}{2}\log x + \frac{1}{2}\log y$

$\therefore \log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$

$\therefore 2\log\left(\frac{x+y}{3}\right) = \log xy$

$\therefore \log\left(\frac{x+y}{3}\right)^2 = \log xy$

$\therefore \left(\frac{x+y}{3}\right)^2 = xy$ [$\because$ Log is one - one Function]

$\therefore \frac{x^2+2xy+y^2}{9} = xy$

$\therefore x^2 + 2xy + y^2 = 9xy$

$\therefore x^2 + y^2 = 9xy - 2xy$

$\therefore \boxed{x^2+y^2 = 7xy}$

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6. If $\log(a-b) = \log 2 + \frac{1}{2}\log a + \frac{1}{2}\log b$ then Prove that $\frac{a}{b} + \frac{b}{a} = 6$.

$\Rightarrow$ Given $\log(a-b) = \log 2 + \frac{1}{2}\log a + \frac{1}{2}\log b$

$\therefore \log(a-b) - \log 2 = \frac{1}{2}\log a + \frac{1}{2}\log b$

$\therefore \log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\therefore 2\log\left(\frac{a-b}{2}\right) = \log ab$

$\therefore \log\left(\frac{a-b}{2}\right)^2 = \log ab$

$\therefore \left(\frac{a-b}{2}\right)^2 = ab$ [$\because$ Log is one-one Function]

$\therefore \frac{a^2 - 2ab + b^2}{4} = ab$

$\therefore a^2 - 2ab + b^2 = 4ab$

$\therefore a^2 + b^2 = 4ab + 2ab$

$\therefore a^2 + b^2 = 6ab$

All terms are divided By ab.

$\therefore \frac{a^2}{ab} + \frac{b^2}{ab} = \frac{6ab}{ab} \Longrightarrow \therefore \boxed{\frac{a}{b} + \frac{b}{a} = 6}$